Chebyshev's Inequality

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The probability that $|X - E(X)| > \varepsilon$ is less than $V(X)/\varepsilon^2$
or the probability that $|X - E(X)| <= \varepsilon$ is more than $1 - V(X)/\varepsilon^2$.
where ε is a positive number.
Proof:

$$V(X) = (x_1 - E(X))^2 p(x_1) + (x_2 - E(X))^2 p(x_2) + \ldots + (x_k - E(X))^2 p(x_k)$$

The x's on the right side of the equation can be numbered so that all of the terms for which $|x_i - E(X)| > \varepsilon$ come before the rest of the terms. Let h be the highest subscript for the terms for which $|x_i - E(X)| > \varepsilon$, then:

$$(x_1 - E(X))^2 p(x_1) + (x_2 - E(X))^2 p(x_2) + \ldots + (x_h - E(X))^2 p(x_h) \leq V(X)$$

Substitute ε^2 for $(x_i - E(X))^2$ in terms on the left side of inequality getting: $\varepsilon^2(p(x_1) + p(x_2) + \ldots + p(x_h)) < V(X)$.

So $p(x_1) + p(x_2) + \ldots + p(x_h) < V(X)/\varepsilon^2$

So since the probability that $|X - E(X)| > \varepsilon$

equals $p(x_1) + p(x_2) + \ldots + p(x_h)$, we get

The probability that $|X - E(X)| > \varepsilon$ is less than $V(X)/\varepsilon^2$
or the probability that $|X - E(X)| <= \varepsilon$ is more than $1 - V(X)/\varepsilon^2$.
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